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1/*f* noise with a low frequency white noise limit

ONE of us (A.A.V.), who had recently been studying channel models for electrical fluctuations caused by potassium transport through nerve membrane, noticed a possible analogy with the flow pattern of grain through an hourglass of unusual shape designed by F. Vaudan, Paris. The hourglass was filled with small grains (0.1 to 1.0 mm diameter) of coloured glass and differed from ordinary hourglasses in the length of the pore connecting the two compartments which was drawn out to about 25 cm; about 80 times the smallest inner diameter (3 mm). When the hourglass was turned, the grain did not flow steadily but exhibited slow irregular density fluctuations (Fig. 1). With a view to the analogy with potassium channels in excitable membranes, we decided to examine comparative power spectra[1-3].

In a series of experiments to measure spectral properties of the grain flow, a laser beam (Spectra-Physics model 155) was sent through the stem of the hourglass and then through a small hole (0.2 mm diameter) placed in front of and close to a photosensitive cell and a dc amplifier.

Light-beam switching resulted in square-waves with a resolution time of about 10^{-4} seconds for the system of cell and amplifier. Another test using a white-noise modulated light source with a white band between 1 and 5,000 Hz showed that the system response was flat within the available range.

In the resulting pulse sequence, most of the pulses had comparable shapes (about triangular, or triangular with clipped peaks when the grains completely blocked the beam during part of their passage) and durations which varied from about 10^{-3} s to 10^{-2} s. After amplification, the power spectrum was calculated (Fig. 2, curve A). Its shape did not change for measurements through different levels of the stem (higher up, where the stream is dense and the light shines through occasionally, or lower down, where individual particles pass through the beam).

The mechanism responsible for the spectral shape must be the interaction between particles and wall and between the particles themselves, while a third friction coefficient is given by the counterflow of air. Clustering is known to occur in bins in the absence of a counterflow of air. The actual values of the friction coefficients (between particles and wall and between the particles themselves) are not of primary importance[4], although they may influence the detailed structure of the spectrum, such as the corner frequencies and the actual slope of the 1/*f* part.

We therefore also examined the spectrum associated with free fall flow out of an open compartment through a gradually narrowing pore (3 mm final diameter) (Fig. 2, curve B). Irregularly shaped, sharp edged steel grit (0.2 mm mean diameter) was used. In both systems the spectrum was white at low frequencies and changed to 1/*f* at higher frequencies. At still higher frequencies the spectrum shape is sensitive to several system parameters and is rather complex.

This is to our knowledge the first report on a physical system with 1/*f* noise which changes at lower frequencies into a different and finite power spectrum. No special tricks need to be invoked, therefore, to circumvent the famous 1/*f* paradox with its explosion of power for frequencies approaching zeros. The mechanism at work here might, for this reason, be of general interest, since noise power spectra are investigated in a diversity of fields such as magnetism, superconductivity, semiconductors, artificial and biological membranes and synapses.

Furthermore, the transition from a 1/*f* spectral density into some other spectral density is not the kind predicted by McWhorter[6]; there one must invoke a Poisson process and a distribution of pulse time constants.

It is not very difficult to explain some of the major features of the measured spectral. These include the low frequency form of the power spectrum (white) and the bandwidth over which the spectrum is essentially 1/*f*. Further, it is possible to show, in general, the conditions under which one may expect deviations from a white spectrum and from this obtain important additional insights into possible source mechanisms.

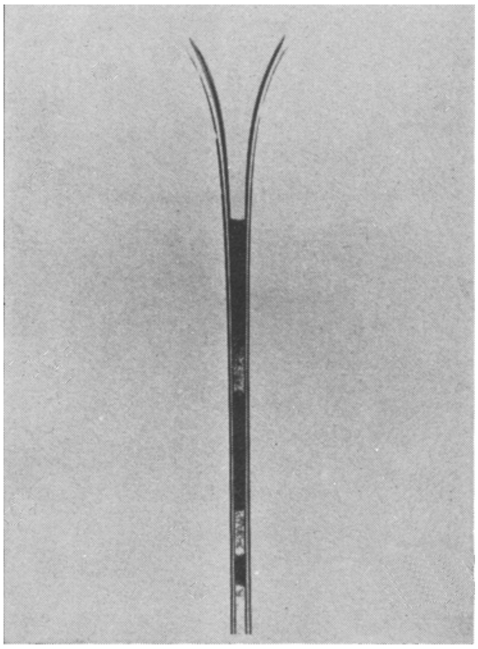

Fig. 1 Flow through hourglass exhibiting clustering (exposure time 1/1000 s).

In order to demonstrate these ideas, let us[7], following Heiden[8], view a noise sequence as a sequence of unitary events (pulses) which may represent current, voltage, magnetic lines, and so on. Each individual pulse has its own time constant, τ, amplitude, h, and shape $y(\tau, i)$. In addition, characterising each pulse is the interval φ between its beginning and the beginning of the next pulse. The simplified sequence can be drawn as in Fig. 3.

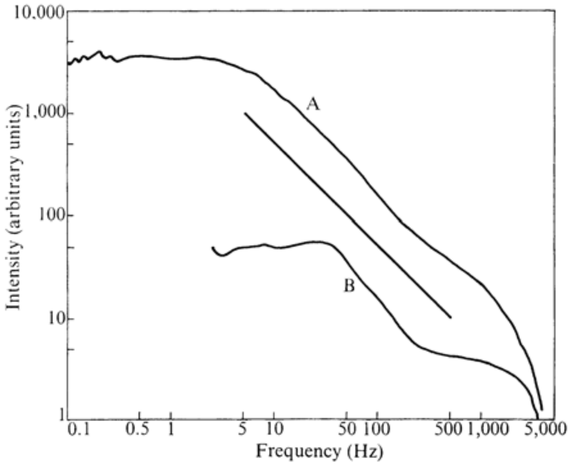

Fig. 2 Power spectra associated with particle flow through hourglass (curve A) and through a narrow pore out of an open compartment (curve B). Line between curves indicates slope of (ideal) 1/*f* noise.

The interval distribution function $\mu(\varphi)$ may be Poisson or non-Poisson. The sequence can be written as

$$I(t) = h_1 y(t,\tau_1) + h_2 y(t-\varphi_1,\tau_2) + h_3 y(t-\varphi_1-\varphi_2,\tau_3) + \dots$$

For those sequences in which φ, τ, and h are functionally independent of one another, the power spectrum is[7]

$$S(f)=N\langle h^2\rangle\langle|F(f,\tau)|\rangle^2\left[1+\frac{2\langle h\rangle^2}{\langle h^2\rangle}\frac{|\langle\ddot{F}(f,\tau)\rangle|^2}{\langle|F(f,\tau)|\rangle^2}\mathrm{Re}\left(\frac{\psi}{1-\psi}\right)\right]$$

where < > indicates average value, N is the mean number of pulses per unit time, $F(f,\tau)$ is the Fourier transform of the individual pulse form $y(t,\tau)$, and

$$\psi = \int_0^\infty \mu(\varphi)\exp(2\pi i f\varphi)d\varphi.$$

For convenience, we can write $S(f) = NA(f)P(f)$ where

$A(f) = \langle h^2\rangle \langle|F(f,\tau)|\rangle^2$ and $P(f)$ is the expression in the brackets.

For Poisson distributions, $\mu(\varphi) = N\exp(-N\varphi)$ and $\mathrm{Re}(\psi/(1-\psi))$ is zero. In these cases, then, the form of the power spectrum is determined by $F(f,\tau)$ and the distribution of pulse time constants, $\nu(\tau)$.

In general,

$$F(f,\tau) = \int_{-\infty}^{+\infty}(t,\tau)\exp(-2\pi i f t)\,dt.$$

At low frequencies ($2\pi f\tau \ll 1$), however,

$$F(f,\tau) = \int_{-\infty}^{+\infty} y(t,\tau)dt.$$

Hence the Fourier transform is frequency independent and the low frequency portion of the spectrum is white[9].

If there is a distribution of time constants, $\nu(\tau)$, among the pulses, then, physically, there exists a maximum τ, τ_m, such that $\nu(\tau)$ is small for $\tau > \tau_m$. Now

$$\langle F(f,\tau)\rangle \approx \int_0^{\tau_m} F(f,\tau)\,\nu(\tau)dt,$$

so at frequencies ($2\pi f\tau_m \ll 1$), $\langle F(f,\tau)\rangle$ is also frequency independent and represents a white spectrum. This is also true for $\langle|F(f,\tau)|\rangle$. Hence, for a sequence of pulses with a distribution of time constants, $A(f)$ is white at low enough frequencies so long as there is no coupling among the parameters of a single pulse.

A similar conclusion obtains for the factor $P(f)$ which incorporates the effect on the power spectrum of deviations from a Poisson sequence.

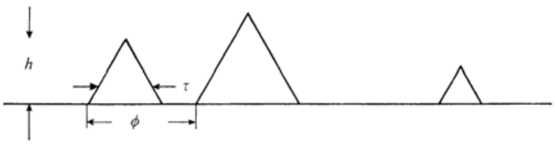

Fig. 3 Pulse sequence parameters.

We have

$$\psi = \int_0^\infty \mu(\varphi)\exp(2\pi i f\varphi)\,d\varphi.$$

Physically there exists a maximum φ, say φ_m, beyond which $\mu(\varphi)$ is arbitrarily small. Hence, for low enough frequencies, ($2\pi f\varphi_m \ll 1$), ψ is frequency independent and, therefore, so is $\mathrm{Re}(\psi/(1-\psi))$.

We therefore have a theorem which says that the power spectrum of any pulse sequence, Poisson or non-Poisson, even with a distribution of pulse time constants, is white at low enough frequencies provided that there is no coupling among the parameters of a single pulse.

There are at least two immediate and useful consequences. First, if there is not too much overlap between pulses, simple visual inspection of the sequence will permit estimates of τ_m and φ_m. This in turn allows an approximate determination of the low frequency bandwidths over which $A(f)$ and $P(f)$ respectively are white. Hence, under these conditions, visual inspection can tell us whether deviations from a white spectrum are due to $A(f)$ or $P(f)$ over a particular bandwidth. This is particularly easy to do in those cases where $\varphi_m > \tau_m$. Secondly, if one should find that at low frequencies ($2\pi f \ll$ the smaller of τ_m^{-1} or (φ_m^{-1}) the spectrum is not white, then the implication is that coupling must exist among the parameters of a single pulse.

The first consequence permits direct comparison between the experimentally determined form of the power spectrum and the theoretical frequency range over which $A(f)$ and $P(f)$ should be white. Since one now knows over what bandwidth the form of the spectrum is due to the shape of the individual pulses, we have a direct way of determining if the sequence is non-Poisson (via $P(f)$). The second allows one to deduce that functional relations (for example, ($\varphi = (\text{constant})\tau$ or $h = \text{constant}/\varphi$) exist between the pulse parameters. We note for example that ($\varphi = (\text{constant})\tau$ is a kind of inhibition where the presence of a pulse with a long time constant tends to delay the appearance of a next pulse while $h = \text{constant}/\varphi$ is a kind of facilitation where a large pulse tends to encourage the appearance of the subsequent pulse. Detailed analysis of the low frequency deviations from a white spectrum will make it possible, in some cases, to select the precise form of the pulse coupling and thereby give further insight into the physical origins of the noise.

If the hourglass flow is examined at different angles with respect to the vertical, the formation of unstable vaults can easily be seen. Clearly, the formation of unstable vaults of different lifetimes generates the clustering effects (Fig. 1) in the flow. In an extensive study (published in Dutch) Peschl[4] investigated the flow of particles from bin openings. A key parameter is the ratio of the opening to the grain diameter. In an hourglass the ratio threshold, 4, is exceeded.

Only unstable vaults occur in normal use and their lifetimes have an upper limit. Hence, the distribution of the intervals between successive particles flowing through a plane must have an upper time limit. In the case of the flow of steel grit through a pore, in our experiments, for example, no intervals longer than 200 ms were found during a period of 10 min with a mean flow of 1,250 grains s^{-1} through the laser beam (0.2 mm diameter). Generally, the individual pulse time constants were considerably less than 20 ms. According to the theorem developed earlier in the paper, the low frequency spectrum should be white for frequencies below about 1/0.2 = 5 Hz as has been found (Fig. 2). The $1/f$ portion of the spectrum is due to the non-Poisson character (clustering) of the individual particles. At higher frequencies a change from $1/f$ to some other spectrum is predicted due to pulse shape, time constant and photocell response. This occurs, as expected, at frequencies >> 100 Hz.

It is tempting to conjecture that in some systems of molecular dimensions with barriers, particles and pores analogous situations might prevail. Molecular size vaults might be formed and thermal motion functions as the agent for the introduction of the instabilities. These systems would exhibit power spectra with strong low frequency contributions (perhaps $1/f$ in form) but, yet, at the very lowest frequencies their spectra would always turn white.

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